

Reduction of entanglement degradation and teleportation improvement in Einstein-Gauss-Bonnet gravity

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Abstract

Bipartite entanglement for states of a non-interacting bosonic or fermionic field in the spacetime of a spherically symmetric black hole of Einstein-Gauss-Bonnet gravity, is investigated. Although the initial state is chosen to be maximally entangled as the Bell states, the Hawking-Unruh effect causes the state to be mixed and the entanglement degrades, but with different asymptotic behaviors for the fermionic and bosonic fields. The Gauss-Bonnet term with positive α can play an anti-gravitation role and so this causes to decrease the Hawking-Unruh effect and consequently reduces the entanglement degradation. On the other hand, the suggested higher dimensions for the spacetime, lead to more entanglement degradation by increasing the dimension. There is a dramatic difference between the behaviors of the entanglement in terms of the radius of the horizon for a five-dimensional black hole and that for higher dimensional black holes.

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I. INTRODUCTION

More and more efforts have been expended on the study of quantum entanglement, which is the essential resource of quantum information processing, in relativistic setting for not only logical completeness but also for the study of the physical bounds of quantum information processing tasks. While Lorentz transformations can not change the overall quantum entanglement of a bipartite state [1], the situation for non inertial observers is different. In order to investigate the observer-dependent character of entanglement, the bipartite entanglement for states of a non-interacting massless scalar field when one of the observers is uniformly accelerated is studied by Alsing and Milburn [2]. They showed that the fidelity of teleportation between two parts in relative acceleration degrades by increasing the acceleration. The bipartite entanglement was also studied by Fuentes-Schuller and Mann, and they showed that only inertial observers agree on the degree of entanglement, and non-inertial observers see a degradation [3]. The acceleration of the observer effectively introduces an environmental decoherence caused by the Unruh-Davis effect. Also, Alsing *et al* discussed the entanglement by different modes of a fermionic field [4]. Their results showed that different types of fields will have qualitative different effects on the degradation of the entanglement. The entanglement by different helicity modes of an electromagnetic field in non inertial reference frame studied in [8].

Some authors extended this issue to the entanglement in curved spacetimes, motivated by the fact that the spacetime near the event horizon of a Schwarzschild black hole resembles Rindler coordinates in the infinite acceleration limit [10]. Ahn showed that for a two-mode squeezed state in a Riemannian geometry, an initial Gaussian state becomes decoherent due to the Hawking effect and in addition a higher squeezing leads to a higher degradation [11]. In this way, the study of entanglement in the black hole geometry is directly related to the black hole information paradox [9]. Hawking radiation in the background of an asymptotically flat static black hole in Einstein gravity is investigated in Ref. [12]. The same issue for sonic black holes is covered in Ref. [14]. Quantum no-cloning theorem for charged black holes is discussed in Ref. [13]. Entanglement in a dynamical spacetime is investigated for bosonic and fermionic fields in Refs. [15] and [16], respectively. The results show that a dynamic metric can generate entanglement and conversely the entanglement encodes information concerning the underlying structure of spacetime. In principle it is possible to fully reconstruct the parameters of the cosmic history from the entanglement entropy. Also, as is shown in Ref. [17], it is possible to generate entanglement by accelerated observers using the Unruh mechanism. Ge extended the teleportation in gravitational fields to the higher dimensional spacetimes such as Schwarzschild and Kerr black hole solutions of the higher dimensional Einstein theory of gravitation [18].

The possibility that spacetime may have more than four dimensions is now a standard assumption in high energy physics [19]. The idea of brane cosmology that is consistent with string theory, suggests that matter and gauge interactions may be localized on a brane embedded into a higher

dimensional spacetime such that the gravitational field can propagate in the whole of the spacetime. In this way we need to consider gravity in dimensions higher than four. In this context one may use another consistent higher dimensional theory of gravity with a more general action, that is the Lovelock theory of gravitation which contains higher powers of Riemann tensor and its derivatives [20]. The first and the second terms in the field equation are the cosmological constant and the Einstein tensor, respectively. The next term, which contains curvature-squared terms is the Gauss-Bonnet tensor. Up to this order, the obtained field equations are called the Einstein-Gauss-Bonnet gravitational field equations. Many Authors have obtained various solutions for these equations by assuming some symmetries for the metric. In Ref. [21] asymptotically AdS solutions of Gauss-Bonnet gravity are obtained without cosmological constant. Also it is shown that an accelerating universe can be obtained from the modified Friedman equation in Gauss-Bonnet gravity [22]. Therefore, it seems that the Gauss-Bonnet term can have an anti-gravitation role.

As a further step in the subject of quantum information in higher dimensional curved spacetimes, we will provide an analysis of quantum entanglement for quantum fields in the spacetime of black hole solution of Einstein-Gauss Bonnet theory. It can be interesting to study how the Hawking temperature can change the entanglement and teleportation in this spacetime. Remembering that the Gauss-Bonnet gravity can have an anti-gravitation role, we expect that in this improved gravitational theory, the quantum information tasks be enhanced. Moreover, we can investigate the effect of higher dimensions on the behavior of the entanglement. The out line of this paper is as follows. In Sec. II, we briefly revisit the Gauss-Bonnet gravity and then review the Hawking-Unruh effect in the black hole solution of this theory. In Sec. III, we set up the problem and calculate the entanglement monotone for both fermionic and bosonic fields. Fidelity of teleportation for a desired observer is derived and explained via appropriate figures in Sec. IV. Some concluding remarks are given in Sec. V.

II. THERMAL DISTRIBUTION OF QUANTUM STATES IN EINSTEIN-GAUSS-BONNET BLACK HOLE

As we know the field equations in Einstein-Gauss-Bonnet gravity can be written as

$$G_{\mu\nu}^{(E)} + \Lambda g_{\mu\nu} + \alpha G_{\mu\nu}^{(GB)} = T_{\mu\nu} \quad (1)$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter, $G_{\mu\nu}^{(E)}$ is the Einstein tensor and $G_{\mu\nu}^{(GB)}$ is the second order Lovelock tensor or Gauss-Bonnet tensor, which is defined in terms of the curvature tensor $R_{\mu\nu\sigma\kappa}$ as

$$\begin{aligned} G_{\mu\nu}^{(GB)} = & 2(R_{\mu\sigma\kappa\tau} R_{\nu}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma} R^{\rho\sigma} - 2R_{\mu\sigma} R^{\sigma}_{\nu} + RR_{\mu\nu}) \\ & - \frac{1}{2} (R_{\mu\nu\sigma\kappa} R^{\mu\nu\sigma\kappa} - 4R_{\mu\nu} R^{\mu\nu} + R^2) g_{\mu\nu}, \end{aligned} \quad (2)$$

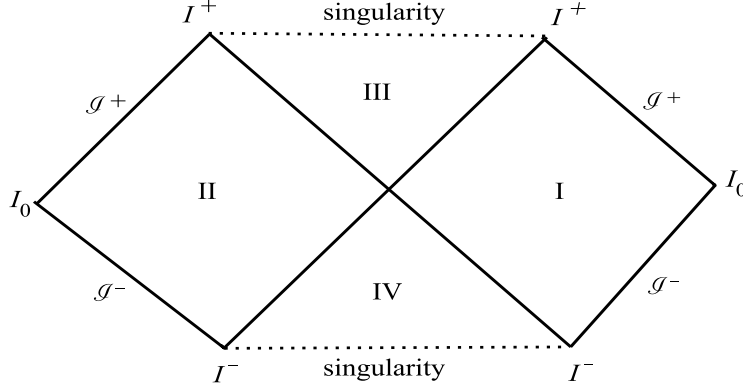


FIG. 1: Penrose diagram for the solution (3).

and α is the Gauss-Bonnet constant which we take it positive. Let us consider a d -dimensional ($d \geq 5$) static spherically symmetric spacetime with the metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-2}^2, \quad (3)$$

where $f(r)$ is an unknown function and $r^2 d\Omega_{d-2}^2$ is the metric of a $(d-2)$ -dimensional subspace. It can be proved that this metric describes a black hole solution of the field equations (1) with $\Lambda = 0$, provided that

$$f(r) = k + \frac{r^2}{2(d-3)(d-4)\alpha} \left(1 \pm \sqrt{1 + \frac{4(d-3)(d-4)\alpha m}{r^{(d-1)}}} \right) \quad (4)$$

where m is the geometrical mass of the black hole and k denotes the curvature of the $(d-2)$ -dimensional subspace [22]. Of course, for the special case of $d = 5$ the function takes a particular form as

$$f(r) = k + \frac{r^2}{4\alpha} \pm \sqrt{\frac{r^4}{16\alpha^2} + \left(|k| + \frac{m}{2\alpha} \right)}, \quad (5)$$

which has a geometrical mass $m + 2\alpha|k|$. Since we are interested in asymptotically flat solutions, we must choose the minus sign and also $k = 1$ in (4) and (5). It is easy to show that in the limit of small α , this $f(r)$ gives the metric of a d -dimensional Schwarzschild solution of Einstein theory, as a requirement of Lovelock gravitation theory. Evidently, the radius of the horizon denoted by r_h can be obtained as the positive root of $f(r)$. For example in the case of $d = 5$ one simply obtains $r_h = \sqrt{m}$.

It is convenient to consider the Penrose diagram for the spacetime (3). Therefore, we invoke the Kruskal coordinates by using the appropriate coordinate transformations. For performing this, we write the metric in (u, v) coordinates as

$$\begin{aligned} ds^2 &= -f(r)dudv + r^2 d\Omega_{d-2}^2, \\ u &= t - r_*, \quad v = t + r_*, \end{aligned} \quad (6)$$

where the Regge-Wheeler tortoise coordinate is defined by $r_* = \int dr/f(r)$. We note that r_* has a logarithmic singularity at the event horizon. This behavior of tortoise coordinate near the event horizon is generally establish for the GR theory [24], and for the Schwarzschild spacetime in this theory, we can explicitly expand r_* around the radius of the horizon. For the spacetime of Gauss-Bonnet black hole, we can still expand r_* in the neighborhood of r_h as

$$r_* \approx \Gamma \ln(r - r_h) + \mathcal{G}(r - r_h) \quad (7)$$

where $\mathcal{G}(r - r_h)$ is a nonsingular function at r_h . Comparing the coefficient Γ with the analogous coefficient in the Schwarzschild spacetime, we infer that this is exactly the inverse of the Hawking temperature of Gauss-Bonnet black hole, that is $\Gamma^{-1} = T$. This temperature can be defined geometrically in terms of the surface gravity of black hole as

$$T = 2\pi \left(-\frac{1}{2} \nabla_\mu \xi_\nu \nabla^\mu \xi^\nu \right)^{-\frac{1}{2}}, \quad (8)$$

which as applied for the spacetime (3), leads to

$$T = \frac{1}{4\pi} \left. \frac{df(r)}{dr} \right|_{r_h}. \quad (9)$$

This as evaluated for (4) and (5), leads to

$$T = \frac{1}{4\pi} \frac{\alpha d^3 - 12\alpha d^2 + 47\alpha d + dr_h^2 - 60\alpha - 3r_h^2}{r_h(2\alpha d^2 - 14\alpha d + 24\alpha + r_h^2)} \quad (10)$$

for the d -dimensional spacetime, and

$$T = \frac{1}{2\pi} \frac{r_h}{(4\alpha + r_h^2)}. \quad (11)$$

for the special case of $d = 5$.

Now we define the Kruskal coordinates

$$U \propto \pm e^{-uT}, \quad V \propto \mp e^{vT} \quad (12)$$

which are used for an analytical extension of the metric. The upper (lower) sign refers to the region I (II) in Fig. (1) which represents the Penrose diagram of the metric (3). Notice that the regions I and II are causally disconnected.

We consider two observers, who are going to communicate through a quantum information protocol in the spacetime of Einstein-Gauss-Bonnet black hole. They share a Bell state, which is composed of two modes of a free quantum field, say ground state and the first excited state of a scalar or a spinor field. These two observers meet each other and share the prescribed quantum state at the asymptotic region of the black hole. Then, one of them, say Alice, stays on a timelike geodesic of the black hole, and consequently is freely falling as an inertial observer. But the other observer, Rob, approaches the event horizon, but he barely accelerates to avoid falling in the black hole. In order to express the entanglement between these observers, we require to construct the quantum field modes as seen by each of observers.

First we consider a massless scalar field ϕ that satisfies the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) = 0. \quad (13)$$

where g denotes the metric determinant. Regarding the spherical symmetry of the metric, ϕ can be separated as

$$\phi(t, r, \Omega) = e^{-i\omega t} \frac{R_{\omega l}(r)}{r^{\frac{d-2}{2}}} Y_{lm}(\Omega), \quad (14)$$

where $Y_{lm}(\Omega)$ is the $(d-2)$ -dimensional spherical harmonic functions and $R_{\omega l}(r)$ satisfies the following equation:

$$\frac{\partial^2 R_{\omega l}}{\partial r_*^2} + \omega^2 R_{\omega l} - f(r) \left(\frac{(d-2)^2}{4r^2} f(r) + \frac{d-2}{2r} \frac{df}{dr} + \frac{l(l+d)}{r^2} \right) R_{\omega l} = 0. \quad (15)$$

The observer Alice who is freely falling into the black hole, sees nothing special at the horizon. Hence she has access to the entire of the spacetime. But as Fig. 1 shows, for the accelerated observer Rob, there are two causally disconnected regions of spacetime denoted by I and II. Since the future directed timelike killing vector corresponding to the region II is directed in the opposite direction of that of the region I, that is $[\partial_t]_I = [-\partial_t]_{II} = [\partial_{-t}]_{II}$, then the positive frequency solutions related to the regions I and II differ up to minus sign in t . Indeed the positive frequency solutions of Eq. (15) are obtained as

$$\begin{aligned} \phi_{I,k} &\sim e^{ikr_* - i\omega t} \equiv e^{i\omega u} \\ \phi_{II,k} &\sim e^{ikr_* + i\omega t} \equiv e^{-i\omega v} \end{aligned} \quad (16)$$

In this way any quantum field can be quantized as

$$\Phi = \sum_{l,m} \int d\omega [(a_I \phi_{I,k} + a_{II} \phi_{II,k}) + H.C.] \quad (17)$$

where $a_{I,k}$ ($a_{II,k}$) and $a_{I,k}^\dagger$ ($a_{II,k}^\dagger$) are the annihilation and creation operators for the mode k in

the region I (II). These operators are called the Schwarzschild operators and satisfy the following relations

$$\begin{aligned} a_{I,k}|0\rangle_{I,k} \otimes |n\rangle_{II} &= a_{II,k}|n\rangle_I \otimes |0\rangle_{II,k} = 0, \\ a_{I,k}^\dagger|0\rangle_{I,k} \otimes |n\rangle_{II} &= |1\rangle_{I,k} \otimes |n\rangle_{II}, \\ a_{II,k}^\dagger|n\rangle_I \otimes |0\rangle_{II} &= |n\rangle_I \otimes |1\rangle_{II,k}. \end{aligned} \quad (18)$$

Since the solutions (16) can not be analytically continued from the region I to the region II, we must express them in the Kruskal coordinates. Regarding Eqs. (12) and (16), we can show that the analytical solutions in the whole of the spacetime can be written as [5]

$$\begin{aligned} \phi_{K,k}^+ &= e^{\frac{\pi\omega}{2T}}\phi_{I,k} + e^{-\frac{\pi\omega}{2T}}\phi_{II,-k}^*, \\ \phi_{K,k}^- &= e^{-\frac{\pi\omega}{2T}}\phi_{I,-k}^* + e^{\frac{\pi\omega}{2T}}\phi_{II,k}, \end{aligned} \quad (19)$$

which correspond to positive and negative frequencies with respect to the Killing vector ∂_U . Instead of (17), one can now expand Φ in terms of $\phi_{K,k}^+$ and $\phi_{K,k}^-$ as

$$\Phi = \sum_{l,m} \int d\omega \left[\left(b_{K,k}^+ \phi_{K,k}^+ + b_{K,k}^- \phi_{K,k}^- \right) + H.C. \right] \quad (20)$$

where

$$\begin{aligned} b_{K,k}^+ &= (\cosh \eta) a_{I,k} - (\sinh \eta) a_{II,-k}^\dagger \\ b_{K,-k}^{-\dagger} &= (\sinh \eta) a_{I,k}^\dagger - (\cosh \eta) a_{II,-k} \end{aligned} \quad (21)$$

where $\tanh \eta = e^{-\pi\omega/T}$. These relations that can be obtained by using Eqs. (19), are just the Bogoliubov transformations between the Schwarzschild operators and the Kruskal operators. The Kruskal vacuum and first excited states for a known k can be expressed as

$$|0\rangle_K = \frac{1}{\cosh \eta} \sum_{n=0}^{\infty} (\tanh^n \eta) |n\rangle_I \otimes |n\rangle_{II}, \quad (22)$$

$$\begin{aligned} |1\rangle_K &= b_{K,k}^+ |0\rangle_K \\ &= \frac{1}{\cosh^2 \eta} \sum_{n=0}^{\infty} (\tanh^n \eta) \sqrt{n+1} |n+1\rangle_I \otimes |n\rangle_{II}. \end{aligned} \quad (23)$$

For a Fermionic quantum field, we can repeat this argument to obtain the appropriate Bogoliubov transformations as

$$\begin{aligned} b_{K,k}^+ &= (\cos \zeta) a_{I,k} - (\sin \zeta) a_{II,-k}^\dagger \\ b_{K,-k}^{-\dagger} &= (\sin \zeta) a_{I,k} + (\cos \zeta) a_{II,-k}^\dagger \end{aligned} \quad (24)$$

where $\tan \zeta = e^{-\pi\omega/T}$. The ground state and the first excited states of the field in Kruskal and Schwarzschild coordinates are related by

$$\begin{aligned} |0\rangle_K &= (\cos \zeta)|0\rangle_{\text{II}} \otimes |0\rangle_{\text{I}} + (\sin \zeta)|1\rangle_{\text{II}} \otimes |1\rangle_{\text{I}} \\ |1\rangle_K &= |1\rangle_{\text{I}}|0\rangle_{\text{II}}. \end{aligned} \quad (25)$$

III. BIPARTITE ENTANGLEMENTS

A. Bosonic entanglement

According to the previous section, we focus on the condition that Rob moves toward the black hole and then stops on a surface outside the event horizon by a slow acceleration, and Alice is freely falling toward the black hole and perhaps after a finite proper time crosses the event horizon. If they share a maximally entangled Bell state far from the black hole as

$$|\phi\rangle_M = \frac{1}{\sqrt{2}} (|0\rangle_M^{\text{A}}|0\rangle_M^{\text{R}} + |1\rangle_M^{\text{A}}|1\rangle_M^{\text{R}}), \quad (26)$$

where the first qubit in each term refers to Alice's cavity and the second qubit refers to Rob's cavity and the index M indicates that the states are considered in the Minkowski spacetime. The state inside the Rob's cavity is no longer perfectly entangled with that of Alice due to the Unruh effect. One can assume that prior to their coincidence, Alice and Rob have not any particle in their cavities and each cavity supports two orthogonal states with the same frequency (single mode approximation), which each is excited to a single particle state Fock state at the coincidence point.

If Rob undergoes a uniform acceleration or stays in a curved spacetime, the state in his cavity must be specified in Schwarzschild coordinates. As a consequence, the second ket in each term of (26) must be expanded according to (22) and (23). We can then rewrite Eq. (26) in terms of Minkowski modes for Alice and Schwarzschild modes for Rob, which leads to a tripartite density matrix as $\rho_{\text{A,I,II}}$. Since Rob is causally disconnected from region II, we must trace over the states in this region, then we obtain a mixed bipartite density matrix denoted as $\rho_{\text{A,I}}$. By rearrangement of elements of the reduced density matrix operator, this can be recast in a block diagonal form as

$$\rho_{\text{A,I}} = \frac{1}{2 \cosh^2 \eta} \sum_n (\tanh^{2n} \eta) \rho_n, \quad (27)$$

where

$$\rho_n = |0, n\rangle\langle 0, n| + \frac{\sqrt{n+1}}{\cosh \eta} |0, n\rangle\langle 1, n+1| + \frac{\sqrt{n+1}}{\cosh \eta} |1, n+1\rangle\langle 0, n| + \frac{n+1}{\cosh^2 \eta} |1, n+1\rangle\langle 1, n+1|, \quad (28)$$

and $|n, m\rangle = |n\rangle_M^{\text{A}}|m\rangle_{\text{I}}^{\text{R}}$. Here the summation goes over all values of n as a consequence of the Bose-Einstein statistics of the scalar field.

The degree of entanglement for the two observers can be quantified by using the the concept of

logarithmic negativity [25] defined as

$$\mathcal{N} = \log_2 ||\rho^T|| \quad (29)$$

where $||\rho^T||$ is the trace norm of the partial transposed matrix ρ^T , which is defined as the sum of the eigenvalues of $\sqrt{(\rho^T)^\dagger \rho^T}$. For a symmetric matrix it can be shown that this is equal to the sum of the absolute value of the eigenvalues of ρ^T . In this case the logarithmic negativity vanishes unless some negative eigenvalues exist. If only one negative eigenvalue N exists, then the logarithmic negativity can be rewritten as

$$\mathcal{N} = \log_2(1 - 2N). \quad (30)$$

The partial transpose of the density operator $\rho_{A,I}$ in Eq. (27) can be obtained by interchanging the Alice's qubit, such that the $(n,n+1)$ block of this partial transposed matrix can be written as

$$\frac{\tanh^{2n} \eta}{2 \cosh^2 \eta} \begin{pmatrix} \tanh^2 \eta & \frac{\sqrt{n+1}}{\cosh \eta} \\ \frac{\sqrt{n+1}}{\cosh \eta} & \tanh^{-2} \eta \frac{n}{\cosh^2 \eta} \end{pmatrix}. \quad (31)$$

Then the negative eigenvalue of $\rho_{A,I}^T$ is obtained as $N = \sum_n N_n$ where

$$N_n = \frac{\tanh^{2n} \eta}{4 \cosh^2 \eta} \left(\frac{n}{\sinh^2 \eta} + \tanh^2 \eta - \sqrt{\left(\frac{n}{\sinh^2 \eta} + \tanh^2 \eta \right)^2 + \frac{4}{\cosh^2 \eta}} \right), \quad (32)$$

is the negative eigenvalue of (31). Now substituting N in Eq. (30), we obtain the logarithmic negativity as an infinite series. This logarithmic negativity is a function of $\eta = \tanh^{-1}(e^{-\pi\omega/T})$ which ranges from 0 to ∞ . For a given ω , the limit $\eta \rightarrow 0$ corresponds to $T = 0$ and as we can see from Eq. (32), this leads to $\mathcal{N} = 1$. On the other hand, the limit $\eta \rightarrow \infty$, which corresponds to infinite Hawking temperature, leads to $\mathcal{N} = 0$ or complete destruction of the entanglement.

The curves in Fig. 2 describe the behavior of the logarithmic negativity for the bosonic entanglement. In Fig. 2(a) the logarithmic negativity is plotted versus the Hawking temperature T . We see that by increasing T , the logarithmic negativity asymptotically descends to zero. The behavior of the logarithmic negativity in terms of the spacetime dimensions d is shown in Fig. 2(b) for some given α . We see that the logarithmic negativity decreases less by increasing α . This behavior is justified by recalling that the positive Gauss-Bonnet coefficient leads to an antigravity effect that can prevent the decoherence. In Fig. 2(c), the α -dependence of entanglement is shown. Only for $d = 5$ the entanglement asymptotically reaches to the unity. In Fig. 2(d), the logarithmic negativity is plotted versus r_h , the horizon radius. While in $d = 5$, in a particular behavior, the curve takes a minimum and return to unity for large radii, for dimensions greater than five, the behavior is monotonic; the entanglement grows from zero at $r_h = 0$ and reaches asymptotically to the unity. This difference can be justified by comparing Eqs. (10) and (11). Especially note that for $r_h = 0$, T in (10) diverges, but T in (11) vanishes.

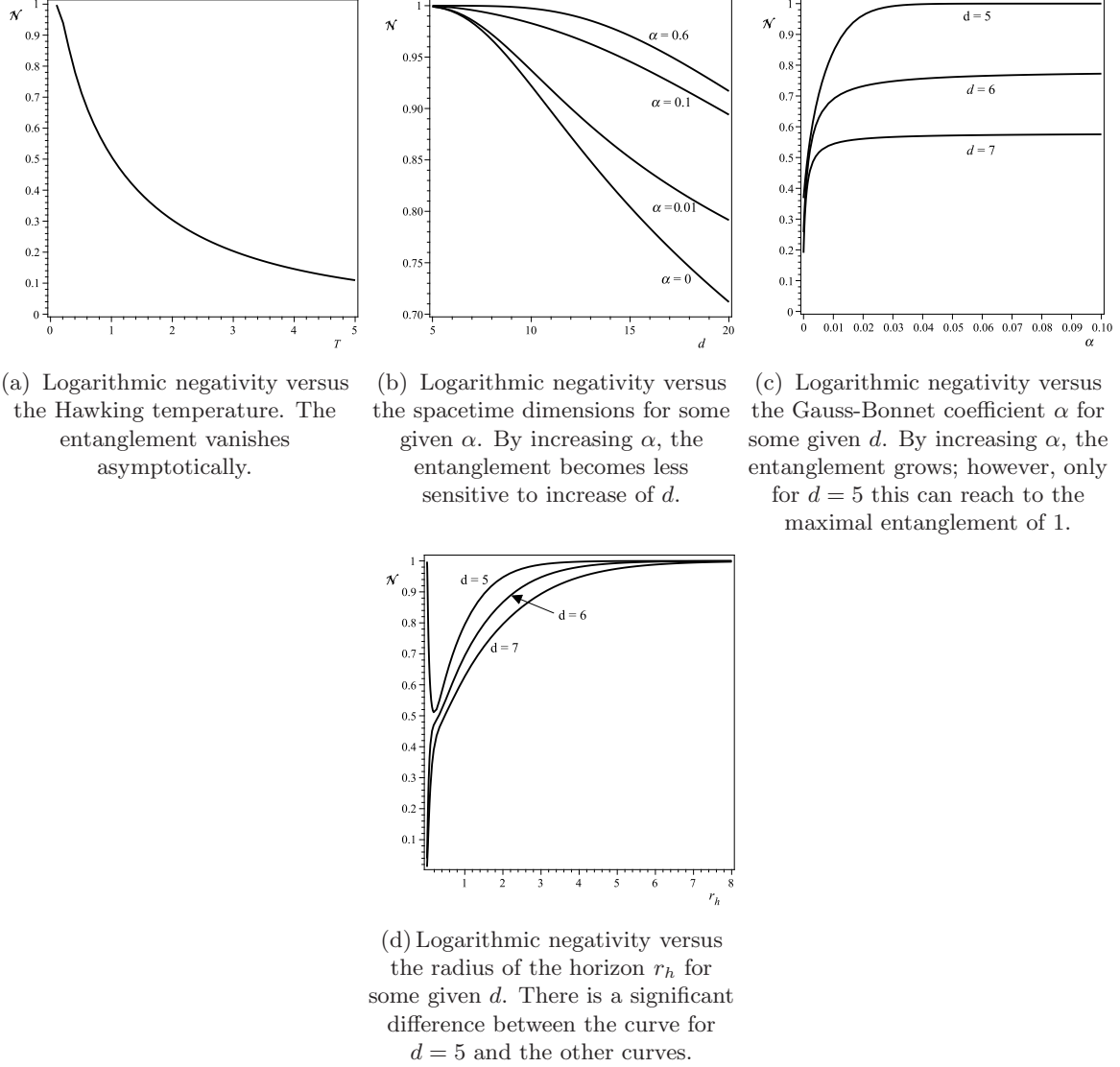


FIG. 2: The logarithmic negativity for the bosonic entanglement.

B. Fermionic entanglement

Suppose that the observers Alice and Rob take the Bell state (26) which this time is built with the states of a Dirac quantum field in the gravitational field. As the bosonic case we assume that two observers are located first far from the black hole horizon, where they share the Bell state. Then Alice falls into the black hole freely, but Rob move toward the event horizon. But he barely, decelerates and keep himself out of the horizon. Using Eq. (24), we can expand the Rob's kets in terms of the Schwarzschild modes. Then, the density operator of the system can be obtained as

$$\begin{aligned} \rho_{A,I,II} = & \frac{1}{2} (\cos^2 \zeta |000\rangle\langle 000| + \sin^2 \zeta |011\rangle\langle 011| + |110\rangle\langle 110|) \\ & + \frac{1}{2} (\cos \zeta \sin \zeta |000\rangle\langle 011| + \cos \zeta |000\rangle\langle 110| + \sin \zeta |011\rangle\langle 110| + H.C.). \end{aligned} \quad (33)$$

This apparently describes a tripartite system. However, Rob is causally disconnected from the region II and so we must trace over the states in that region, which results a mixed density matrix denoted as $\rho_{A,I}$. Then we obtain the partially transposed of the resulting matrix as

$$\rho_{A,I}^T = \frac{1}{2} \begin{pmatrix} \cos^2 \zeta & 0 & 0 & 0 \\ 0 & \sin^2 \zeta & \cos \zeta & 0 \\ 0 & \cos \zeta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (34)$$

This has one negative eigenvalue $-\frac{\cos^2 \zeta}{2}$ which obviously leads to a logarithmic negativity as

$$\mathcal{N} = \log_2 (1 + \cos^2 \zeta). \quad (35)$$

Since $\tan \zeta = e^{-\pi\omega/T}$, ζ ranges from 0 to $\frac{\pi}{4}$, then \mathcal{N} can take only values between ~ 0.58 and 1. This means that the fermionic entanglement cannot be erased completely even for an infinite Hawking temperature.

In Fig. 3 we have plotted this logarithmic negativity versus T, d, α and r_h . These curves are comparable with the curves in Fig. 2. The general behaviors of the corresponding curves are the same; however, they differ in details. Especially note to the difference between the asymptotic or starting values of the curves in the corresponding figures. It seems that the fermionic entanglement is generally robust than the bosonic entanglement.

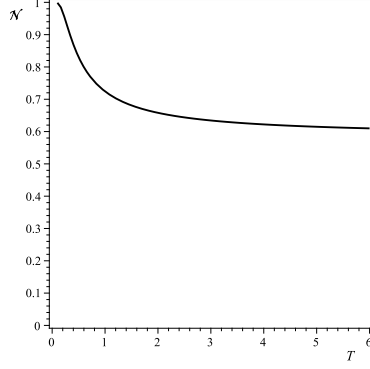
IV. FIDELITY OF TELEPORTATION

A. Teleportation by the bosonic field

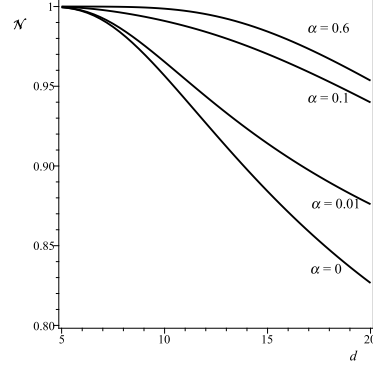
We set up the problem for performing a teleportation between the observers Alice and Rob. Suppose that each cavity supports two orthogonal modes with the same frequency labelled A_i and R_i with $i = 1, 2$, such that each mode can be excited to a single photon Fock state at the coincidence point. The state held by Alice and Rob is chosen to be the entangled Bell state:

$$|\Phi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_R + |1\rangle_A |1\rangle_R),$$

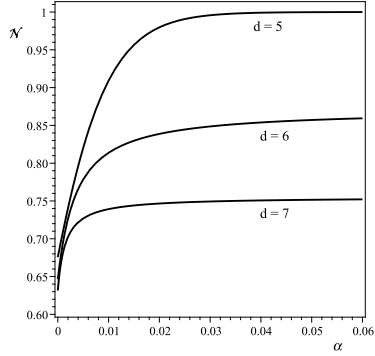
where $|0\rangle_A, |1\rangle_A$ are defined in terms of the physical Fock states for Alice's cavity by the dual-rail basis states as $|0\rangle_A = |1\rangle_{A_1} |0\rangle_{A_2}$, $|1\rangle_A = |0\rangle_{A_1} |1\rangle_{A_2}$, with the similar expressions for the Rob's cavity. Teleportation procedure provides a way to teleport an unknown state $|\Psi\rangle_A = \alpha|0\rangle + \beta|1\rangle$ to Rob, utilizing quantum entanglement. We should assume that Alice has an additional cavity, which contains a single qubit with dual-rail encoding by a photon excitation of a two-mode vacuum state. This will allow Alice to perform a joint measurement on the two orthogonal modes of each cavity mandated with her. After Alice's measurement, Rob becomes aware of this by a classical channel. Rob's photon will be projected according to the measurement outcome. The final state that Rob receives can be given by $|\varphi_{ij}\rangle = x_{ij}|0\rangle + y_{ij}|1\rangle$, where $(x_{00}, y_{00}) = (\alpha, \beta)$, $(x_{01}, y_{01}) =$



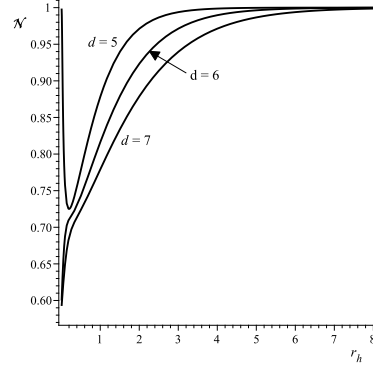
(a) Logarithmic negativity versus the Hawking temperature. The entanglement never reaches to the values less than ~ 0.58 .



(b) Logarithmic negativity versus the spacetime dimensions for some given α . By increasing α , the entanglement becomes less sensitive to increase of d .



(c) Logarithmic negativity versus the Gauss-Bonnet coefficient α for some given d . By increasing α , the entanglement grows; however, only for $d = 5$ this can reach to the maximal entanglement of 1.



(d) Logarithmic negativity versus the radius of the horizon r_h for some given d . There is a significant difference between the curve for $d = 5$ and the other curves.

FIG. 3: The logarithmic negativity for the fermionic entanglement.

$(\beta, \alpha), (x_{10}, y_{10}) = (\alpha, -\beta)$, and $(x_{11}, y_{11}) = (-\beta, \alpha)$, are four possible conditional state amplitudes. Once receiving the Alice's result of measurement, Rob can apply a unitary transformation to verify the protocol in his local frame. However, Rob must notice the fact that his cavity will become teemed with thermally excited photons because of the Hawking-Unruh effect and his state is mixed. When Alice sends the result of her measurement to Rob, if Alice has not yet cross the future horizon, the state that Rob observes must be traced out over the region II, and the density state reduces to

$$\begin{aligned}
\rho_{ij}^{(I)} &= \text{Tr}_{II}(|\varphi_{ij}\rangle_M \langle \varphi_{ij}|) = \sum_{n=0}^{\infty} p_n \rho_{ij,n}^I \\
&= \frac{1}{\cosh^6 \eta} \sum_{n=0}^{\infty} \sum_{m=0}^n ((\tanh^2 \eta)^{n-1} [(n-m)|x_{ij}|^2 + m|y_{ij}|^2] \times |m, n-m\rangle_I \langle m, n-m| \\
&\quad + (x_{ij} y_{ij}^* \tanh^{2n} \eta \sqrt{(m+1)(n-m+1)}) \times |m, n-m+1\rangle_I \langle m+1, n-m| + \text{H.C.}),
\end{aligned} \tag{36}$$

where p_n denotes the expansion coefficient given as

$$p_0 = 0, \quad p_1 = 1/\cosh^6 \eta, \quad p_n = \frac{(\tanh^2 \eta)^{n-1}}{\cosh^6 \eta}, \quad (37)$$

and $|m, n - m\rangle$ denotes a state of n total excitations in the region I.

Suppose that upon receiving the result (i, j) of Alice's measurement, Rob can apply the rotation operators restricted to the 1-excitation sector of his state as $Z_I^i X_I^j|_{n=1}$. The fidelity of Rob's final state with the state that Alice attempts to teleport to him is given by:

$$F \equiv \text{Tr}_I (|\Psi\rangle_I \langle \Psi| \rho^I) = \cosh^{-6} \eta \quad (38)$$

In Fig. (4), we have plotted this fidelity in terms of T , α and d . We see that by increasing T , the fidelity rapidly falls to zero leading to a fatal error in the teleportation. This means that the teleportation in this case is very sensitive to the Hawking temperature. Also, it is seen that the fidelity decreases in terms of spacetime dimension d , that is, additional dimensions lead to more errors in the teleportation. The fidelity increases by increasing α , as a consequence of antigravity role of α .

B. Teleportation by the fermionic field

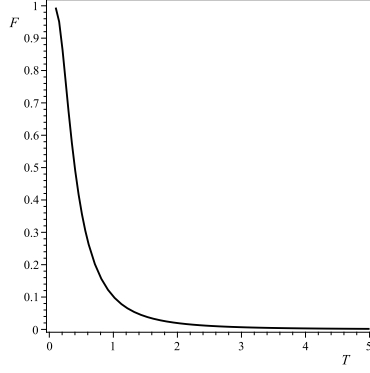
Using dual-rail basis as an excitation of a spin-up state in one of two possible modes in Alice and Rob cavities, one obtains the fidelity of Rob's final state as

$$F = \cos^2 \zeta$$

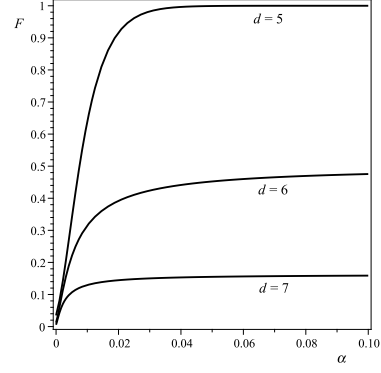
In Fig. (5) we have plotted this fidelity versus T , α and d . In this case the fidelity never reaches to values less than 0.5. Although the general behavior of the fermionic fidelity is similar to the bosonic fidelity, but the fidelity in this case can not be destroyed completely, that is, in teleportation by the fermionic field less errors occur. Again we see that, in the Gauss-Bonnet gravitation theory the teleportation can be improved.

V. CONCLUSION

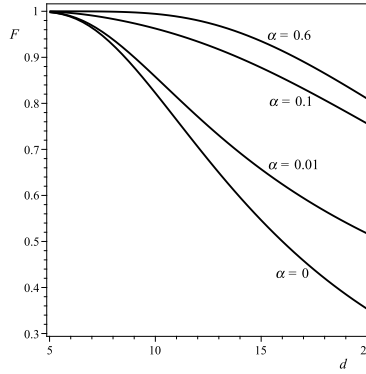
In this work, we studied the bipartite entanglement between the states of a non-interacting bosonic or a fermionic field in the spacetime of a d -dimensional spherically symmetric black hole of the Einstein-Gauss-Bonnet gravitation. We considered two observers; one of them, say Alice, was freely falling into the black hole and so used an inertial frame described by the Kruskal coordinates. The other observer, say Rob, accelerated to avoid falling into the black hole, and so he was in a non-inertial frame described by Schwarzschild-like coordinates. The Bogoliubov transformations that relate the states in the inertial frame to the non-inertial frame, were calculated by comparing the problem with its Schwarzschild analogue. We assumed that the observers



(a) Fidelity of teleportation versus the hawking temperature. The curve rapidly drops to zero.



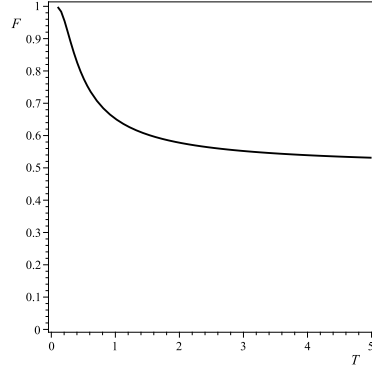
(b) Fidelity of teleportation versus the Gauss-Bonnet coefficient α for some given d . Increase of α improves the teleportation.



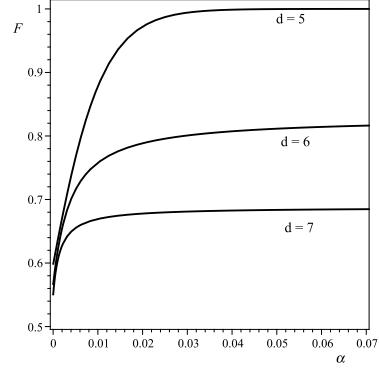
(c) Fidelity of teleportation versus the spacetime dimensions d for some given α . Additional dimensions lead to more errors in the teleportation.

FIG. 4: The fidelity of teleportation by the bosonic field.

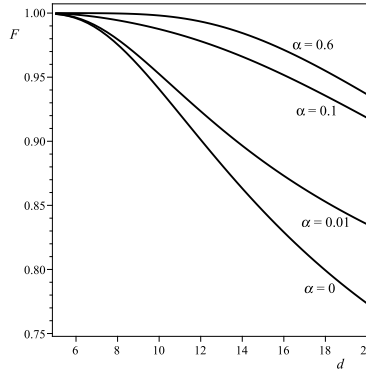
initially share a Bell state built by single the mode states of the fields. We investigated the logarithmic negativity a measure for the entanglement. Although general behavior of this bipartite entanglement is similar to the behavior of the entanglement in the Schwarzschild spacetime or entanglement in the accelerated Rindler frames; however, some new features emerge here. In particular, the Gauss-Bonnet coefficient α , which determines the strength of the higher derivative terms in the gravitation theory, and also the suggested higher dimensions for the spacetime, lead to important results. The Gauss-Bonnet term with positive α can play an antigravity role in the cosmological context. From the viewpoint of quantum information theory this causes to decrease the Hawking-Unruh effect and consequently reduces the entanglement degradation. We showed that by increasing α , the logarithmic negativity saturated to values depending on the dimension of the spacetime; only for $d = 5$ this saturated to the unity. Also, we studied the effect of higher dimensions on the entanglement degradation. By increasing the dimensions of the spacetime more entanglement degradation occurs. Moreover, we investigated the behavior of the logarithmic negativity in terms of the radius of the horizon. For $d = 5$, the logarithmic negativity starts from 1 at $r_h = 0$, takes a minimum at a given r_h , and finally return to 1. However, in other dimensions



(a) Fidelity of teleportation versus the hawking temperature. The curve never reaches to values less than 0.5.



(b) Fidelity of teleportation versus the Gauss-Bonnet coefficient α for some given d . Increase of α improves the teleportation.



(c) Fidelity of teleportation versus the spacetime dimensions d for some given α . Additional dimensions lead to more errors in the teleportation.

FIG. 5: The fidelity of teleportation by the fermionic field.

this grows uniformly from zero (for the bosonic field), or a nonzero value (for the fermionic field), to the unity.

As an application of these results, we discussed the teleportation between Alice and Rob, who use modes of the bosonic or the fermionic field. We calculated the fidelity as a quantitative measure of the accuracy of the information transmission. Expectedly, the behavior of the fidelity is in complete agreement with the behavior of the logarithmic negativity.

A possible extension of this work includes the black hole solutions of higher order Lovelock gravitation theory. Then we encounter more parameters that affect on the quantum information process.

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